

Smooth Origami Animation by Crease Line Adjustment

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Introduction

A folding process of simple origami can be represented by a sequence of simple folding processes [Miyazaki et al. 1996], but this method cannot be applied for complex origami models because the folds in a complex model are constrained around vertices and not independent of each other.

Hence the rigid origami model that represents origami as plates connected by hinges is suitable for complex origami models. Balkcom [2002] shows a way for calculating rigid origami kinematics. This model has two problems, lack of degrees of freedom and a singularity problem that some of the crease lines cannot begin to be folded until other crease lines are completely folded flat.

We propose a method for making a smooth and comprehensible origami animation by avoiding above-mentioned problems by adding and adjusting crease lines on an origami model. The overall folding process is constructed from crease line pattern.

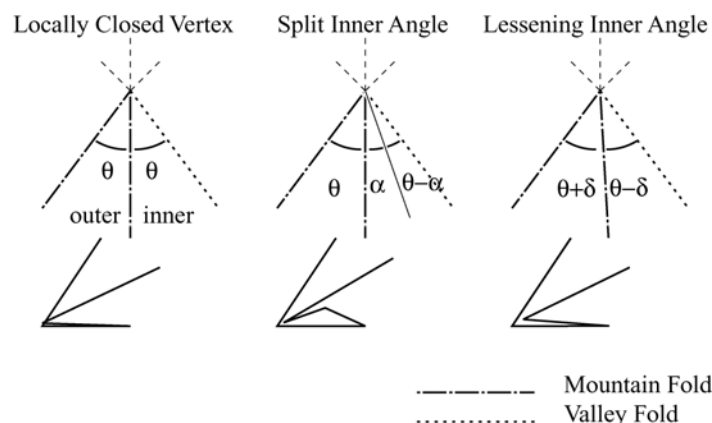
Method

Rigid Origami Kinematics

We use rigid origami model, which is plate-hinge model that contains a closed loop around each vertex, for the simulation of origami. Change in configuration is simulated by calculating pseudo inverse of a global constraint matrix, which is built from conditions for single vertex rigid origami that Belcastro and Hull [2002] presented.

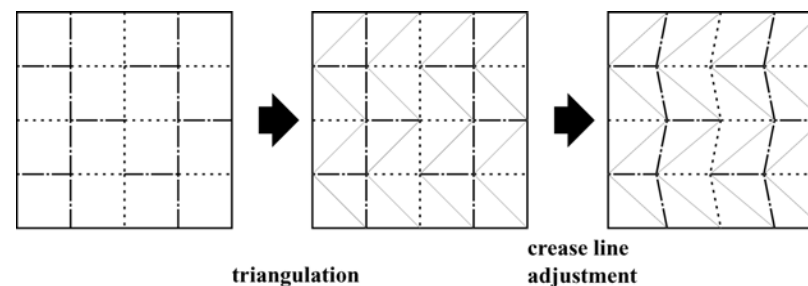
Locally Closed Vertex

We call a vertex "locally closed" if there is a set of three adjacent crease lines from the vertex that the outer two are symmetrical about the center crease and their mountain-valley attributes are opposite. Locally closed vertex is not rigidly foldable before the center crease is completely folded flat. We call the angle between the center crease line and one of the other two crease lines with the same mountain-valley attribute as the center crease line "outer angle", and the other angle "inner angle." We can make a locally closed vertex "open" by splitting or by lessening the inner angle.



Overview

Our method makes locally closed vertices "open" by Triangulation of Polygons and Crease Line Adjustment.



Triangulation of Polygons

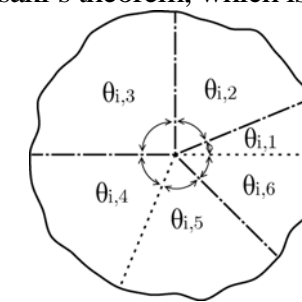
Most origami models are not rigidly foldable because of the lack of degrees of freedom. The degrees of freedom are $m-3n$ if the constraint matrix is full rank, where m and n are the number of edges and vertices inside the paper respectively. By adding $k-3$ crease lines on polygons with k vertices ($k>3$), we can add $k-3$ degrees of freedom to the model. By this triangulation, some of the models become rigidly foldable.

We chose the crease lines for the triangulation that split as many inner angles of locally closed vertices as possible to avoid singularity of the folds. Because the added crease lines must not intersect each other, it is not always possible to split the inner angles of the locally closed vertices.

Crease Line Adjustment

Another way of making locally closed vertices "open" is to adjust the angle between the crease lines. Positions of the vertices are moved so that the inner angle of each locally closed vertex gets smaller than the outer angle. For appropriate deformation through rigid origami simulation, keeping of the flat-foldability of originally flat-foldable vertices is necessary. A condition for single vertex flat-foldability is given by Kawasaki's theorem, which is,

$$\sum_{k=1}^{ni/2} \theta_{i,2k} = \sum_{k=1}^{ni/2} \theta_{i,2k-1} = \pi$$



where $\theta_{i,1}, \dots, \theta_{i,ni}$ are the angles between adjacent crease lines around the vertex i .

Global constraint matrix is given by

$$\begin{bmatrix} \frac{\partial F_i}{\partial X} & \frac{\partial F_i}{\partial Y} \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = [0] \text{ where}$$

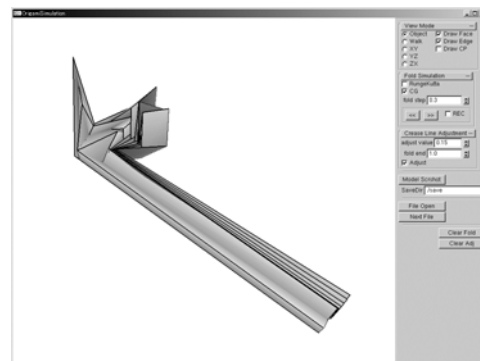
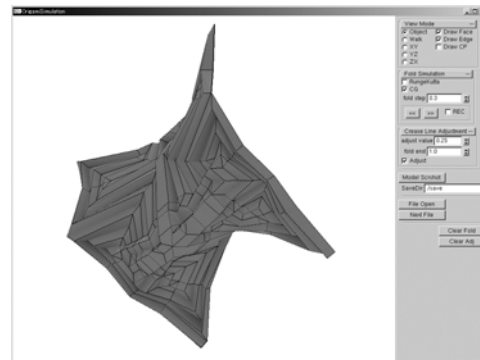
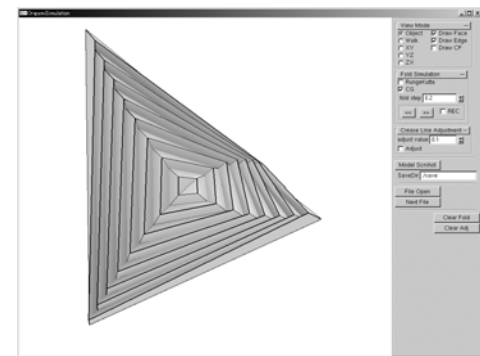
$$F_i(X, Y) = \sum_{k=1}^{ni/2} \theta_{i,2k} - \sum_{k=1}^{ni/2} \theta_{i,2k-1} = 0$$

Under this constraint, positions of the vertices (X, Y) are adjusted to make the inner angle smaller than the outer angle for each locally closed vertex. Although any finite angle adjustment to the right direction can avoid the local singularity, the progress of folding of crease line is better balanced when the crease line adjustment angle is larger. We have observed through simulation of actual models that the animation is smooth and the configuration of the origami model is easily comprehensible if the overall folding process is well balanced.

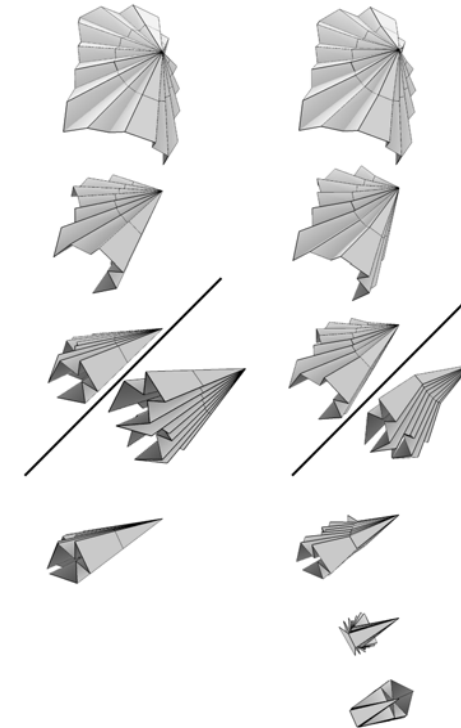
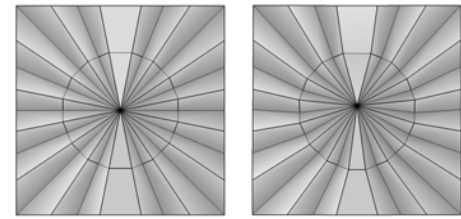
For making the folding process balanced, we change the adjustment angle for each vertex according to the "imbalance" of the creases around the vertex. The adjustment angle is a monotonic function of a variable $\{max(crease_angle) - min(crease_angle)\}$. This results in no adjustment for both unfolded and flat-folded state of origami.

Result

We have implemented this method as an interactive application and applied for complex origami models. The proposed method enabled the simulation of many of the origami models, and could make a smooth folding animation from crease pattern to folded shape. There are still some origami models that cannot be folded because of the lack of degrees of freedom and the globally closed structure.



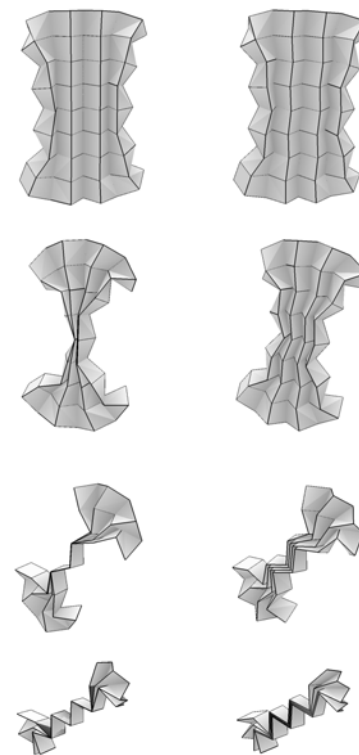
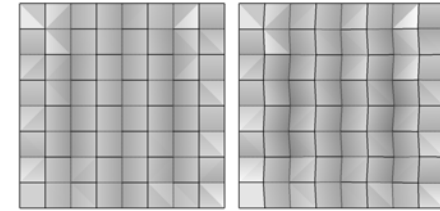
No Adjustment With Adjustment



(Both Triangulated)

A folding process of fold lines over other folds is modeled naturally with crease line adjustment.

No Adjustment With Adjustment



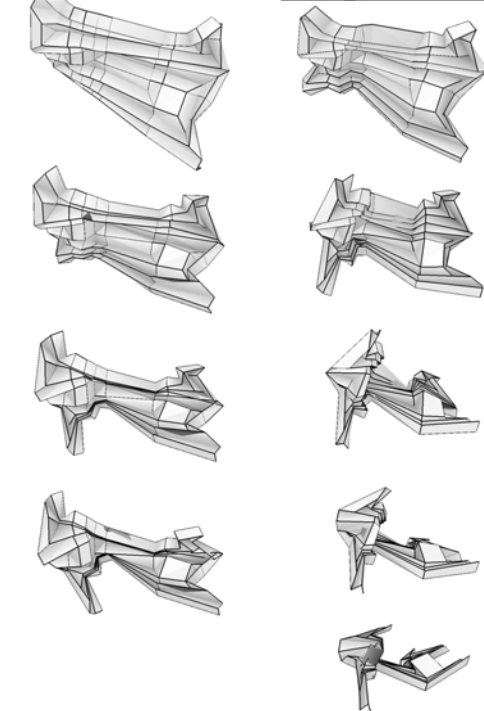
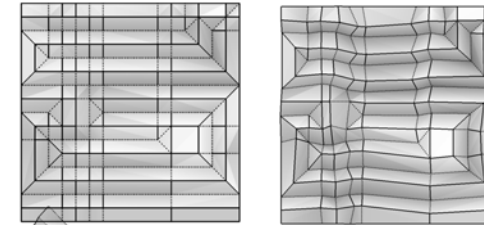
(Both Triangulated)

Just triangulation of squares resulted in smooth animation of "double accordion folding."

By adjusting the crease lines, the configuration gets comprehensible by displacement of the overlapping edges.

The adjusted crease pattern is identical to the crease pattern of "Miura Fold."

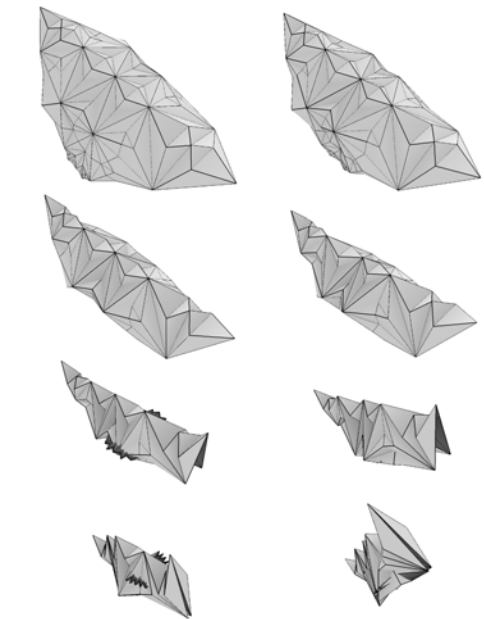
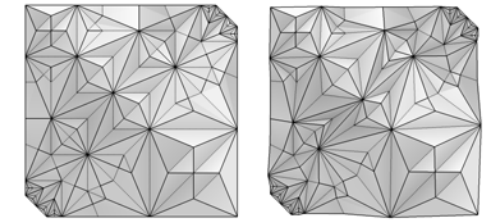
No Adjustment With Adjustment



(Both Triangulated)

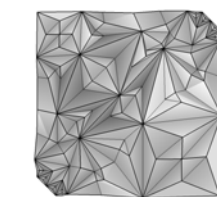
The animation of the folding process of a complex model as "Teapot and Teacup" is possible by triangulation of the polygons and crease line adjustment.

No Adjustment With Adjustment



(Both Triangulated)
(Model: "Devil" by Jun Maekawa)

This model cannot be folded completely flat without intersection of the polygons and too much deformation of the model to be perceived as the same shape as the original.



(Completely folded, but with intersection and too much deformation)

Reference

- BALKCOM, D. 2002. *Robotic Origami Folding*. PhD thesis, Carnegie Mellon University.
- BELCASTRO, S. M. and HULL, T. C. 2002. A Mathematical Model for Non-Flat Origami. In *Proceedings of the 3rd International Meeting of Origami Mathematics, Science and Education*, 39-52.
- MIYAZAKI, S., YASUDA, T., YOKOI, S., and TORIWAKI, J. 1996. An Origami Playing Simulator in the Virtual Space. *The Journal of Visualization and Computer Animation*, 7, 1, 25-4